



---

## Exploiting Explicit and Implicit Structure in Complex Optimization Problems

Robert Mifflin  
WASHINGTON STATE UNIVERSITY

---

09/24/2014  
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory  
AF Office Of Scientific Research (AFOSR)/ RTA  
Arlington, Virginia 22203  
Air Force Materiel Command

<b>REPORT DOCUMENTATION PAGE</b>			<i>Form Approved</i> <b>OMB No. 0704-0188</b>	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. <b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</b>				
<b>1. REPORT DATE (DD-MM-YYYY)</b> 14-09-2014		<b>2. REPORT TYPE</b> FINAL PERFORMANCE REPORT		<b>3. DATES COVERED (From - To)</b> JUNE 2011-JUNE 2014
<b>4. TITLE AND SUBTITLE</b> EXPLOITING EXPLICIT AND IMPLICIT STRUCTURE IN COMPLEX OPTIMIZATION PROBLEMS			<b>5a. CONTRACT NUMBER</b>	
			<b>5b. GRANT NUMBER</b> FA9550-11-1-0139	
			<b>5c. PROGRAM ELEMENT NUMBER</b>	
<b>6. AUTHOR(S)</b> MIFFLIN, ROBERT B.			<b>5d. PROJECT NUMBER</b>	
			<b>5e. TASK NUMBER</b>	
			<b>5f. WORK UNIT NUMBER</b>	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> WASHINGTON STATE UNIVERSITY 240 FRENCH ADMINISTRATION BLDG. PULLMAN WA 99164-0001			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> AIR FORCE OFFICE OF SCIENTIFIC RESEARCH 875 NORTH RANDOLPH ST., ROOM 3112 ARLINGTON VA 22203-1768			<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> AFOSR	
			<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>	
<b>12. DISTRIBUTION / AVAILABILITY STATEMENT</b>  DISTRIBUTION A- APPROVED FOR PUBLIC RELEASE				
<b>13. SUPPLEMENTARY NOTES</b>				
<b>14. ABSTRACT</b> A quasi-Newton version of a VU-bundle algorithm for minimizing a convex function, with knowledge of only one subgradient value at each point, was perfected to the point where numerical superlinear convergence could be observed. The algorithm is important, because it is the type needed for minimizing implicitly defined functions resulting from applying decomposition, relaxation and/or dualization techniques to complex real-world optimization problems. Also, valuable research was carried out for nonconvex objective functions. This included a non-VU bundle method for composite functions where the outer function is a positively homogeneous convex function and the inner vector function is a smooth mapping. Such an explicitly known structure separates the two difficulties of nonconvexity and nonsmoothness by allowing only the components of the inner mapping to be nonconvex and only the outer function to be nonsmooth. This new algorithm was shown to be convergent to stationary points and judged to be the best performer out of four methods tested on many examples. Also, significant progress was made on designing a VU algorithm to run on general semismooth functions. This entailed making a V-model based bundle method subalgorithm with convergence to stationary points.				
<b>15. SUBJECT TERMS</b> NONSMOOTH OPTIMIZATION VU ALGORITHM BUNDLE METHODS				
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>  SAR	<b>18. NUMBER OF PAGES</b>  3
<b>a. REPORT</b> U	<b>b. ABSTRACT</b> U	<b>c. THIS PAGE</b> U		
				<b>19b. TELEPHONE NUMBER (include area code)</b> 509 335 3153

Principal Investigator: Robert Mifflin, mifflin@math.wsu.edu, 509-335-3153

Grant/Contract Title: Exploiting Explicit and Implicit Structure in Complex Optimization Problems

Grant/Contract Number: FA9550-11-1-0139

Program Manager: Dr. Fariba Fahroo

Reporting Period: 16 June 2011 to 15 June 2014

#### Accomplishments:

Decomposition techniques are often the best choice for solving large scale complex optimization problems. Applications of such techniques in energy planning, generation and distribution are contained in grant supported publications [11], [13], [14], [3] and [5]. The decomposition type of approach involves minimizing a nonsmooth objective function with special structural properties. For (more general) equilibrium problems, similar ideas can be exploited, as explained in publications [5] and [4], dealing with structured variational inequalities resulting from generalized Nash equilibrium problems.

For optimization problems subject to uncertainty, further decomposition methods and structural properties of nonsmooth constrained convex problems are exploited in [1], [7], [10], [14], [3] and [2]. In particular, publication [14] considers a special bundle method, which appears to be computationally effective in chance-constrained programming for optimal management of a set of cascaded hydro-reservoirs. An earlier method that uses inexact objective information for two-stage stochastic programming problems appears in [7]. These methods can be embedded in a more general framework described in [9] and extended in [8] to level bundle variants capable of handling inexact information.

For nonsmooth objective functions, possibly nonconvex, paper [12] develops a computationally effective proximal bundle method for minimizing a composite function where the inner mapping is smooth and the outer function is a positively homogeneous convex function of several variables. Such an explicitly known structure separates the two difficulties of nonconvexity and nonsmoothness by allowing only the outer function to be nonsmooth and only the components of the inner mapping to be nonconvex. This new algorithm was shown to be convergent to Clarke stationary points and judged to be the best performer among four methods tested on several numerical examples.

During the last grant year significant progress was made towards the goal of producing a foundation for designing a future VU-type minimization algorithm to run on semismooth locally Lipschitz functions for which only one Clarke generalized gradient can be computed at a point. Paper [6] gives an excellent illustration of superlinear convergence attained by a VU-algorithm running on a convex objective example. The intent of current work is to attain this fast type of asymptotic convergence if the proposed algorithm gets close enough to a Clarke stationary point that is a strong local minimizer.

The groundwork for the nonconvex case involved development of a 'V-model' based bundle method subalgorithm that has provable convergence to stationary points and can make adequate estimates of 'V-subspace' bases in the presence of nonconvexity. This entailed adding safeguarded second order correction terms to the polyhedral affine subfunction expressions as done in the superlinearly convergent algorithm for the single variable case in [R. Mifflin, Math. Prog. 28 (1984) 50-71].

This work has also produced a very large class of convex model functions for use in bundle methods to obtain asymptotic stationarity for nonconvex objectives. This class contains much more than polyhedral functions, for example, the composite model in [12].

These new research results will be contained in a submission to a special edition of the Journal of Convex Analysis dedicated to the memory of Jean Jacques Moreau, a pioneer in the development of convex analysis.

Publications during reporting period:

[1] V. Guigues and C. Sagastizabal, Exploiting structure of autoregressive processes in chance-constrained multistage stochastic linear programs, Operations Research Letters 40 (2012) 478-483.

[2] V. Guigues and C. Sagastizabal, Risk averse feasible policies for large-scale multistage stochastic linear programs, Mathematical Programming 138 (2013) 167-198.

[3] V. Guigues and C. Sagastizabal and J. Zubelli, Robust management and pricing of LNG contracts with cancellation options, Journal Optimization Theory and Applications 161(1) (2014) 179-198.

[4] J.P. Luna, C. Sagastizabal and M. Solodov, A class of Dantzig--Wolfe type decomposition methods for variational inequality problems, Mathematical Programming 143 (2014) 177-209.

[5] J.P. Luna, C. Sagastizabal and M. Solodov, Complementarity and game-theoretical models for equilibria in energy markets: Book Chapter deterministic and risk-averse formulations, Handbook of Risk Management for Energy Production and Trading, R.M. Kovacevic, G.Ch. Pflug, and M.T. Vespucci (editors), Springer, International Series in Operations Research and Management Science, Vol. 199, Chapter 10, pp. 237-264, 2014.

[6] R. Mifflin and C. Sagastizabal, A Science Fiction Story in Nonsmooth Optimization Originating at IIASA, pp. 291-300 in Optimization Stories, extra volume ISMP edited by M. Grotschel, Documenta Mathematica, Bielefeld, 2012.

[7] W. Oliveira and C. Sagastizabal, and S. Scheimberg, Inexact Bundle Methods for Two-Stage Stochastic Programming, SIAM Journal on Optimization 21 (2011) 517-544.

[8] W. Oliveira and C. Sagastizabal, Level bundle methods for oracles with on-demand accuracy, Optimization Methods and Software, published online 14 Feb 2014.

[9] W. Oliveira and C. Sagastizabal, Bundle methods in the XXIst century: A bird's-eye view, accepted for publication in Pesquisa Operacional (special issue "Continuous Optimization"), 2014.

[10] C. Sagastizabal, Nonsmooth optimization: Thinking outside of the black box, SIAG/OPT Views-and-News 22(2) (2011) 2-9.

[11] C. Sagastizabal, Divide to conquer: decomposition methods for energy optimization, Mathematical Programming 134(1) (2012) 187-222.

[12] C. Sagastizábal, Composite Proximal Bundle Method, Mathematical Programming 140(1) (2014) 189-233.

[13] C. Sagastizabal and M.V. Solodov, Solving generation expansion planning problems with environmental constraints by a bundle method, Computational Management Science 2003 (2012)(9) 163-182.

[14] W. Van Ackooij and C. Sagastizábal, Constrained Bundle Methods for Upper Inexact Oracles with Application to Joint Chance Constrained Energy Problems, SIAM Journal on Optimization 24(2) (2014) 733-765.